

Loughborough University
Department of Mathematical Sciences
MATHEMATICAL CHALLENGE
CHRISTMAS - 2011

Problem 1. Find the 2011-digit number such that its ratio to the sum of its digits is minimal. Justify your answer.

What is the answer for an N -digit number for arbitrary N ?

Problem 2. Show that an infinite arithmetic progression

$$x_n = x_0 + nd, \quad n = 0, 1, 2, \dots$$

with non-zero difference d contains an infinite geometric progression if and only if the ratio x_0/d is rational. Find such a geometric progression for

$$x_n = \frac{1}{2011} + \frac{5}{2012}n, \quad n = 0, 1, 2, \dots$$

Problem 3. Batman and Catman play the following game. On a table there are 257 cards with numbers $0, 1, 2, \dots, 256$ written on them. Batman chooses any 128 of them and takes them away. After that Catman removes any 64 cards from the remaining pack. Then Batman takes away any 32 cards and then Catman does the same with 16 cards of his choice, etc. In his last (which is his 4-th) move Catman takes away only one card, leaving two cards on the table. The difference between two numbers is the amount of golden ducats Catman must pay to Batman. Obviously Batman would like to make it as large as possible, while Catman is keen to minimise it.

What will this amount be if both play with optimal strategy? Justify your answer.

Remarks.

1. There will be a first prize of £50 to the person handing in what will be considered to be the best effort to these problems. There may also be special prizes for the most original solutions.
2. Any student registered on one of the undergraduate programmes in the School of Mathematics may submit solutions to any or all of these problems.
3. Prof. A. P. Veselov and Dr. B. Winn will be the judges for the solutions to these problems, and their decision will be final.
4. Solutions should be handed to any of the judges on or before Friday 27 January 2012.